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ROBUST AND OPTIMAL MIXED H_2/H_∞ CONTROL FOR ACTIVE MAGNETIC BEARING SYSTEMS

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Abstract:
This paper presents an algorithm for designing a robust and optimal controller for active magnetic bearing systems. The active magnetic bearing systems are widely applied for high speed machining due to no contact operation, low fiction, lubrication-free operation, and extended life. However, they are non-linear, unstable, multiple input and multiple output systems. Therefore, a robust and optimal controller is required. In this paper, we derive dynamic equation and analyze response of the open loop system. Based on the dynamic response of the system, we propose a suitable controller with robust and optimal criteria using particle swarm optimization. The simulation results show that the closed loop system attains good performance in compared with conventional PID controllers.
Keywords: *Robust controller; mixed H∞/H2 controller; particle swarm optimization; magnetic bearing systems; MIMO systems.*

1. Introduction

Active Magnetic Bearings (AMBs) have many advantages over conventional bearings such as contact-less operation, low frictional losses, lubrication-free operation, and extended life. With these advantages, AMBs are widely applied in many real applications for replacing conventional bearings, including high speed machining [1], or magnetically supported flywheels [2,3]. However, AMB systems are non-linear due to the nonlinearities of electromagnetic field. They are also inherently open loop unstable systems. To adapt with the dynamic changing of ABMs systems, adaptive control techniques were applied for ABMs systems such as nonlinear adaptive inverse control by Jeng J. T. [4]; linear parameter varying control by Lu B. et al. [5]; and self-tuning fuzzy PID control by Chen K. U. [6]. In Jeng's research, he used a nonlinear adaptive inverse controller to overcome the nonsystematic design of nonlinear ABMs systems. He used a neural network structure for his adaptive inverse controller. However, this method required a long training time of inverse model with neural network and cannot work on-line.

In recent years, robust H_∞ control has received a great deal of attention because of its systematic design methodology with robust stability guaranteed. For ABMs systems, robust gain scheduling H_∞ control was previously proposed by Matsumura et al. to eliminate unbalance vibration [7]. In this design they separated the systems into two simple SISO systems, therefore, the coupling effect among axis was not considered. Jastrjebski R. P. et al. [8] introduced a centralized H_∞ control method with a procedure for selecting weighting functions using genetic algorithm (GA). Another robust control technique was also presented in a research by Sheu J. S. et al. [9]. These above H_∞ control designs showed that the closed loop systems were robustly stable but the order of received controllers are very high. In Matsumura design [7], the controller has 32 state variables.

Mixed H_2/H_∞ control is a newly robust and optimal control technique for systems associated with sources of uncertainties and external disturbances, firstly proposed by Bernstein and Haddad [10]. This control technique then has been extendedly studied by many researchers [11,12]. However, the main limitation of the mixed H_2/H_∞ control is that it is high order; hence it is complicate to implement in reality both by hardware or software solutions with higher cost and more complicated in compared with simple conventional PID controllers. Therefore, a low-order mixed H_2/H_∞ control is preferred for embedded controllers. The low-order mixed H_2/H_∞ control is defined as finding a low-order controller satisfied mixed H_2/H_∞ robust and optimal criteria. However, this design comes up with a non-convex and complicate optimization problem which cannot be solved by convex optimization tools such as gradient-based or linear programming methods.

This research proposes a systematic methodology for designing a robust and optimal mixed H_2/H_∞ controller for ABM systems. This design methodology satisfies the criteria of mixed H_2/H_∞ paradigm using particle swarm optimization approach. The design methodology also investigation into the thorough understanding of the ABMs dynamics system so that a cross-feedback scheme with a tracking notch filter are used.

2. Methods

2.1. Dynamics Model of Active Magnetic Bearing Systems

2.1.1. Active Magnetic Bearing Systems

Consider a vertical ABM system with a flywheel at the center is shown in Fig.1. The rotor is considered to be rigid because the first bending frequency of rotor is about 1200Hz (72.000 rpm) which is far above the nominal operating speed (40.000 rpm). The radial motion of the vertical ABM system is controlled by two magnetic bearings A and B.

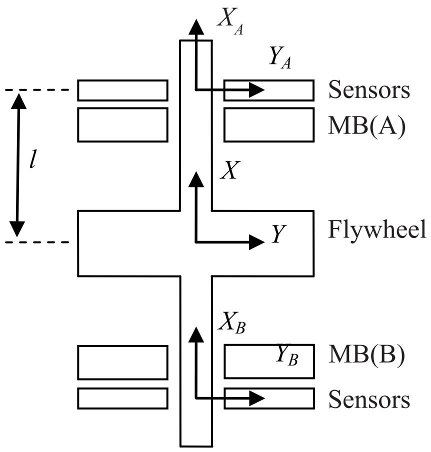

Figure 1. *Vertical ABMs Systems*

The dynamic equations can be written in a matrix form as follows

$$\begin{bmatrix} I_t & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & I_t & 0 \\ 0 & 0 & 0 & m \end{bmatrix}\begin{bmatrix} \ddot{\theta}_y \\ \ddot{z} \\ \ddot{\theta}_y \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} 0 & 0 & I_p\Omega & 0 \\ 0 & 0 & 0 & 0 \\ -I_p\Omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{\theta}_y \\ \dot{z} \\ \dot{\theta}_y \\ \dot{y} \end{bmatrix} +$$
$$+\begin{bmatrix} 2k_p l^2 & 0 & 0 & 0 \\ 0 & 2k_p & 0 & 0 \\ 0 & 0 & 2k_p l^2 & 0 \\ 0 & 0 & 0 & 2k_p \end{bmatrix}\begin{bmatrix} \theta_y \\ z \\ \theta_y \\ y \end{bmatrix}$$
$$= \begin{bmatrix} -lk_i & lk_i & 0 & 0 \\ k_i & k_i & 0 & 0 \\ 0 & 0 & lk_i & -lk_i \\ 0 & 0 & k_i & k_i \end{bmatrix}\begin{bmatrix} i_{za} \\ i_{zb} \\ i_{ya} \\ i_{yb} \end{bmatrix} + \qquad (1)$$
$$+\begin{bmatrix} l & l & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -l & l \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} me\Omega^2 \sin(\Omega t + \varphi) \\ me\Omega^2 \sin(\Omega t) \\ me\Omega^2 \cos(\Omega t + \varphi) \\ me\Omega^2 \cos(\Omega t) \end{bmatrix}$$

The equation (1) can be expressed in state space variable form as follows

$$\begin{bmatrix} \underline{\ddot{r}} \\ \underline{\dot{r}} \end{bmatrix} = \begin{bmatrix} -\underline{\tilde{M}^{-1}}\,\underline{\tilde{\Gamma}} & -\underline{\tilde{M}^{-1}}\,\underline{\tilde{K}} \\ \underline{I} & \underline{0} \end{bmatrix}\begin{bmatrix} \underline{\dot{r}} \\ \underline{r} \end{bmatrix} + \begin{bmatrix} \underline{\tilde{M}^{-1}}\,\underline{B} \\ \underline{0} \end{bmatrix}\underline{i} + \begin{bmatrix} \underline{\tilde{M}^{-1}}\,\underline{E} \\ \underline{0} \end{bmatrix}\underline{w} \qquad (2)$$

Let $X_p = \begin{bmatrix} \underline{\dot{r}} \\ \underline{r} \end{bmatrix}$, $Y_{pa} = \begin{bmatrix} \underline{i} \\ \underline{0} \end{bmatrix}$, and $\underline{F} = \begin{bmatrix} \underline{w} \\ \underline{0} \end{bmatrix}$

Then the state space of the ABM system becomes:

$$\begin{cases} \underline{\dot{X}}_p = A_p \underline{X}_p + \underline{B}_p \underline{Y}_{pa} + \underline{\Lambda}_d \underline{F} \\ \underline{Y}_p = \underline{C}_p \underline{X}_p \end{cases} \qquad (3)$$

Let the transfer function from input to output is $\tilde{G}_p(s)$ without present of imbalance disturbance, and the transfer function from imbalance disturbance to output is $H_p(s)$, we have:

$$\tilde{G}_p(s) = \frac{\underline{Y}_p(s)}{\underline{Y}_{pa}(s)} = \underline{C}_p(s\underline{I} - \underline{A}_p)^{-1}\underline{B}_p$$
$$H_p(s) = \frac{\underline{Y}_p(s)}{\underline{F}(s)} = \underline{C}_p(s\underline{I} - \underline{A}_p)^{-1}\underline{\Lambda}_d \qquad (4)$$

Then $\underline{Y}_p(s) = \tilde{G}_p(s)\,\underline{Y}_{pa}(s) + H_p(s)\,\underline{F}(s)$ (5)

where $\tilde{G}_p(s)$ and $H_p(s)$ are four-by-four transfer function matrixes.

2.1.2. Dynamic Model of Power Amplifier

The power amplifier transforms controller output voltages to currents that flow through magnetic bearing coils. Pulse width modulation is normally used for ABMs systems. A simplified first order transfer function is used to represent power amplifier dynamics

$$G_{PA}(s) = \frac{K_{PA}}{Ls + R} I_{4\text{x}4} \qquad (6)$$

2.2. Robust and Optimal Mixed H_2/H_∞ Control

2.2.1. Robust and Optimal Mixed H_2/H_∞ Control Setup

To control for robust stability of an ABM system with optimal control energy and tracking error, three main problems must be addressed in a closed loop system:

(i) The coupling effect by gyroscopic moment among axes of the bearings must be controlled. This is implemented by a symmetric controller with cross-feedback term.

(ii) The imbalance disturbance torques which depends on rotating speed must be canceled out. This is implemented by adding a tracking notch filter.

(iii) The system must be robustly stable with all uncertainties of the plant and present of sensor noise and control energy as well as tracking error is

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minimized. This is implemented with mixed H_2/H_∞ control framework.

Because matrix $\underline{A}_p$ in (4) is changed with rotation speed Ω, $\tilde{G}_p(s)$ is also changed with Ω. This changing can be represented as structured multiplicative uncertainty in the following form

$$\tilde{G}_p(s) = G_p(s)(I + \Delta G_p(s)) \quad (7)$$

Where $G_p(s)$ is nominal model at working speed Ω and $\triangle G_p(s)$ is uncertainty due to changing of Ω. Due to this symmetric property of the ABMs systems, then a symmetric controller be applied. The closed loop control system setup for mixed H_2/H_∞ controller design is presented in Fig. 2. A source of noise in measurement is run-out, called $S_s(s)$, created by out of roundness of the shaft surface and must be considered in the control system design.

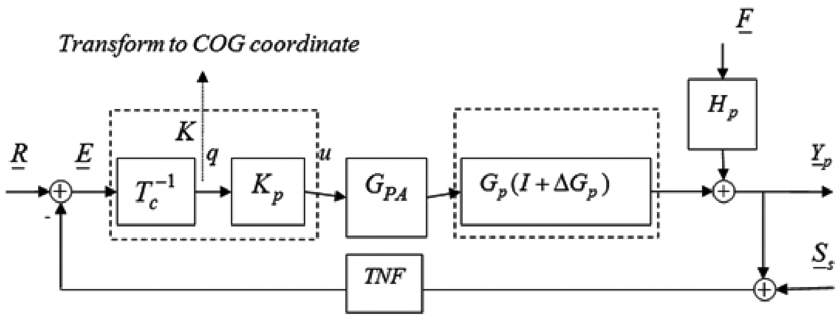


Figure 2. *Closed loop control diagram with power amplifier dynamics, system uncertainty, imbalance disturbance and sensor noise*

In the control diagram in Fig. 2, we have

$$\begin{bmatrix} i_{za} \\ i_{zb} \\ i_{ya} \\ i_{yb} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \begin{bmatrix} \theta_y \\ z \\ \theta_z \\ y \end{bmatrix} \text{ or } u = K_p q \quad (8)$$

$$K(s) = K_p(s) . T_c^{-1} \quad (9)$$

Based on the analysis of ABMs dynamic systems with open loop step response, a multi-variable, center of mass co-ordinate PID control law (without integral term) with cross-feedback control is proposed as follows.

$$\begin{cases} i_{za} + i_{zb} = -K_{pz} z - K_{dz} \dot{z} \\ i_{za} - i_{zb} = -K_{p\theta y} \theta_y - K_{d\theta y} \dot{\theta}_y - K_{p\theta z} \theta_z + K_{d\theta z} \dot{\theta}_z \\ i_{ya} + i_{yb} = -K_{py} y - K_{dy} \dot{y} \\ i_{ya} - i_{yb} = -K_{p\theta z} \theta z - K_{d\theta z} \dot{\theta}_z + K_{p\theta y} \theta y - K_{d\theta y} \dot{\theta} y \end{cases} \quad (10)$$

Rewrite equation (10), we have the controller $K_p(s)$ is:

$$K_P(s) = \begin{bmatrix} -K_{PD1} & K_{PD2} & -K_{PD3} & 0 \\ K_{PD1} & K_{PD2} & K_{PD3} & 0 \\ -K_{PD3} & 0 & -K_{PD1} & K_{PD2} \\ K_{PD3} & 0 & K_{PD1} & K_{PD2} \end{bmatrix} \quad (11)$$

Where each PID controllers with low pass filter attached is selected in the form

$$K_{PDi} = \frac{1}{(\tau_{lp} s + 1)^2}(K_{Pi} + K_{Di} s), \ \text{i} = 1 \div 3 \quad (12)$$

$K_{p1}(s)$ is sub-controller for controlling bearing *A* and bearing *B*, and $K_{p2}(s)$ is sub-controller with cross-feedback term. $K_{p2}(s)$ is used to cancel the coupling gyroscopic moment of the system. With the multiple inputs multiple outputs (MIMO) controlled system as shown in Fig. 3, the uncertainty of the plant is modeled by multiplicative uncertainty. Where $G_p(s)$ is the nominal plant model, $\triangle G_p(s)$ is the structured uncertainty of the plant, $K_p(s)$ is a PID-type controller, $\underline{R}_p(s)$ is reference input, $\underline{F}(s)$ is imbalance disturbance, and $\underline{S}_s(s)$ is sensor noise. We have:

$$\Delta G_p(s) = \left(\frac{\tilde{G}_p(s)}{G_p(s)} - I \right) \quad (13)$$

Assume that the plant perturbation, $\triangle G_p(s)$, is upper bound by a stable weighting function $W_1(s)$ with

$$\|\Delta G_p(s)\|_\infty \le \|W_1(s)\|_\infty \quad (14)$$

And $K(s) = K_p(s) . T_c^{-1}$ (15)

It is proved by small gain theorem by Bernstein, D.S. et al. [10] that if a controller, *K*(*s*), is designed so that:

(i) The nominal closed-loop system ($\triangle G_p(s)$ = 0) is asymptotically stable.

(ii) The robust stability performance against plant uncertainty satisfies the following inequality

$$J_{\infty,a} = \|W_1(s) T(s)\|_\infty < 1 \quad (16)$$

(iii) The robust stability performance against external disturbance (sensor noises) satisfies the following inequality

$$J_{\infty,b} = \|W_2(s)S(s)\|_\infty < 1 \quad (17)$$

Then, the closed-loop system is also asymptotically stable with $\triangle G_p(s)$, and sensor noises $\underline{S}_s(s)$, where $W_2(s)$ is an upper bound stable weighting function of $\underline{S}_s(s)$, $S(s)$ and $T(s)$ are sensitivity and complementary sensitivity functions of the system, respectively.

$$S(s) = (I + G_p(s)K(s))^{-1} \quad (18)$$

$$T(s) = G_p(s)K(s)(I + G_p(s)K(s))^{-1} \quad (19)$$

In many control systems, not only the robust stability against plant uncertainty and sensor noise, but also small tracking error and small control energy are also important. The problem of minimizing the tracking error and control energy of a system can be defined as minimizing the following multi-objective cost function.

$$\begin{cases} J = \beta\|E(s)\|_2^2 + \alpha\|U(s)\|_2^2 = \\ = \dfrac{\beta}{2\pi}\int_0^\infty trace\left(E(j\omega)E(j\omega)^T\right)d\omega += \\ = \dfrac{\alpha}{2\pi}\int_0^\infty trace\left(U(j\omega)U(j\omega)^T\right)d\omega \\ \beta + \alpha = 1 \end{cases} \quad (20)$$

where $E(s) = \left(I + G_p(s)K(s)\right)^{-1}R(s)$ (21)

$$U(s) = \left(I + G_p(s)K(s)\right)^{-1}R(s)K(s) \quad (22)$$

The robust and optimal control problem of ABMs system is defining as finding all parameters of the controller in equation (12) which minimizes the cost function in (21) subject to constraints (17) and (18). This is a non-convex optimization problem with constraints, then PSO is proposed to solve this optimization problem.

2.2.2. Selecting of Weighting Functions

Weighting function W_1 is selected as a bound function of $\triangle G_p$. The nominal model is derived at nominal speed Ω = 40.000 *rpm*, assume that the changing speed of rotor is 25%. Then we can calculate $\Delta G_{p1}, \Delta G_{p2}, \Delta G_{p3}$ at three speed Ω = 20.000 *rpm*, 30.000 *rpm*, and 50.000 *rpm* using formula $\Delta G_p = \left(\dfrac{\tilde{G}_p}{G_p} - I\right)$, bode plot of these uncertainties and the weighting function W_1 is selected as a bound is show in Figure 3.

The weighting function is selected as follows

$$W_1(s) = \frac{10^2(s - 10^7)^2}{(s - 10^4)^2} I_{4x4} \quad (23)$$

The weighting function W_2 is selected as a bound function of sensor noises (run-out).

$$W_2(s) = \frac{k_{si}}{\tau_s s + 1} I_{4x4} \quad (24)$$

where k_{si} is sensor gain, $\tau_s = 1/(2\pi f_s)$.

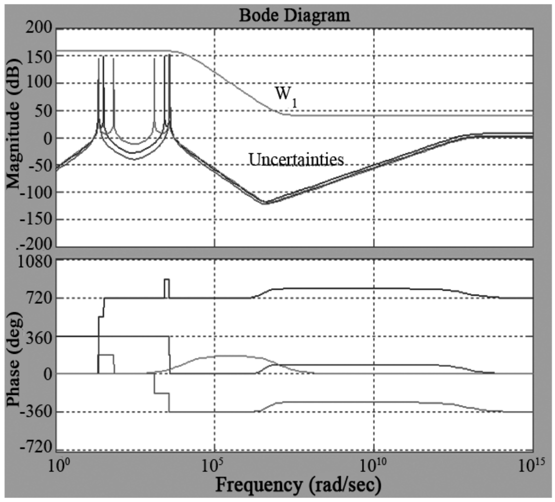


Figure 3. *Bode plot of the weighting function W_1 and singular value uncertainties*

2.2.3. Particle Swarm Optimization Algorithm

PSO algorithm is one of the most recent evolutionary techniques. The method was developed by simulation of simplified social model, where each population is called a swarm. In PSO algorithm, multiple solutions are together and collaborate simultaneously. Each candidate, called a particle, flies through problem space to look for the optimal position, similar to food searching of bird swarm. A particle adapts its position based on its own knowledge, and knowledge of neighboring particles. The algorithm is initialized with a population of random particles. It searches for the optimal solution by updating particles in generations.

Let the search space be *N*-dimensional, then the particle *i* is represented by an *N*-dimensional position vector, $x_i = (x_{i1}, x_{i2},..., x_{iN})$. The velocity is represented also by an *N*-dimensional velocity vector, $v_i = (v_{i1}, v_{i2}, ..., v_{iN})$. The fitness of particles is evaluated by the objective function of the optimization problem. The best previously visited position of particle i is noted as its individual best position, $P_i = (p_{i1}, p_{i2},..., p_{iN})$. The position of the best individual of the whole swarm is noted as the global best position, $G = (G_1, G_2,..., G_N)$. At each step of searching process, the velocity of particle and its new position are updated according to the following two equations [17].

$$v_i(k+1) = w\,.\,v_i(k) + c_1\,.\,r_1\,.\,(P_i(k) - x_i(k)) + \\ + c_2\,.\,r_2\,.\,(G(k) - x_i(k)) \quad (25)$$

$$x_i(k+1) = x_i(k) + v_i(k) \quad (26)$$

where w, called inertia weight, controls the impact of previous velocity of the particle. r_1, r_2, are random variables in the range of [0,1]. c_1, c_2 are positive constant parameters called acceleration coefficients. The value of each component in v is limited to the range [$-v_{max}$, v_{max}] to control excessive roaming of particles outside the search space.

2.2.4. Designing Robust and Optimal Mixed H_2/H_∞ Controller Based on PSO

We select PID-type controller as shown in (26), then each particle of the swarm has 6 parameters as θ = {K_{p1}, K_{d1}, K_{p2}, K_{d2}, K_{p3}, K_{d3}}. The step of PSO algorithm for searching the PD controller parameters is as follows:

Step 1: Set the particle i to θ = {K_{p1}, K_{d1}, K_{p2}, K_{d2}, K_{p3}, K_{d3}}. The number of parameters of the controller is the particle dimension. The maximum number of iterations is defined as GenMax.

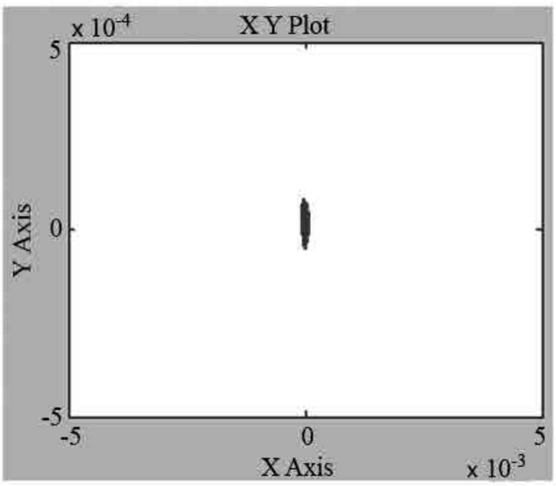
Step 2: When the swarm size is H, initialize a random swarm of H particles as $\{\theta_1, \theta_2, ..., \theta_H\}$.

Step 3: For each generation of particles, evaluate the objective function for each particle using the objective function shown by equation (20), constraints (21) and (22), and determine the individual best P_{ik} and global best $G(k)$.

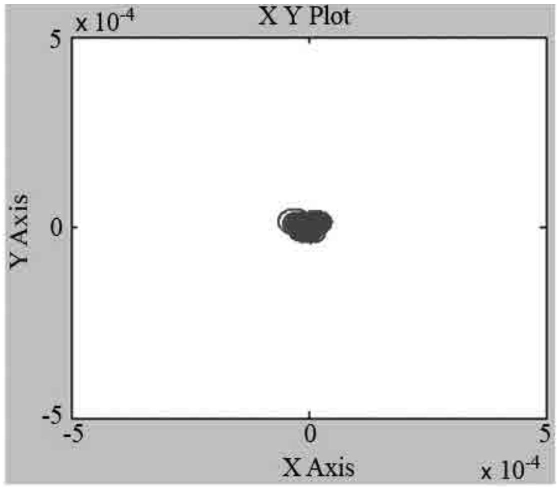
Step 4: Update the particle velocity and its new position using the equations (25) and (26).

Step 5: When the maximum number of iterations is obtained, the algorithm is ended. If the maximum number of iterations is not obtained, go back to Step 3.

3. Results and Discussion



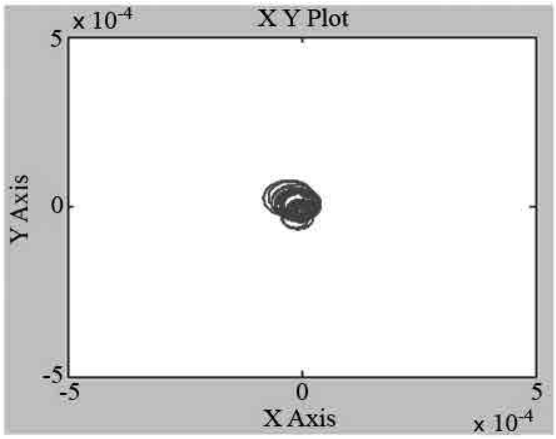
a. X_a vs Y_a



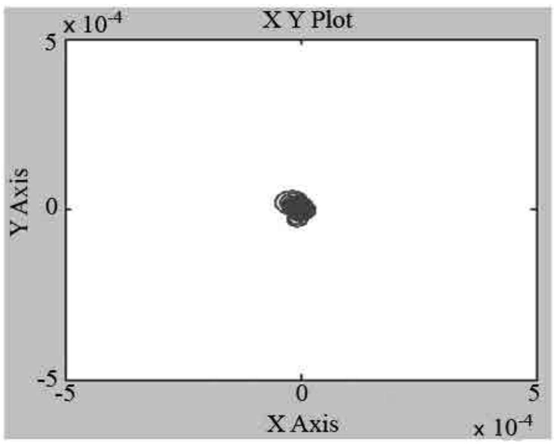
b. X_b vs Y_b

Figure 4. *Plot of positions for A and B bea rings with robust and optimal controller*

To compare with the conventional design, we have the simulation of the conventional design as shown in Fig. 5.



c. X_a vs Y_a



d. X_b vs Y_b

Figure 5. *Plot of positions for A and B bearings with conventional design*

The following parameters of swarm are set-up: swarm size $H = 10$, dimension of each particle is 6, $c_1 = c_2 = 2$, numbers of iterations is 200, the initial weight w is 0.95 and changing to final weight is 0.4, velocity limit is set to [-200, 200], $\beta = 0.8$ and $\alpha = 0.2$. The program is running in MATLAB with 10 times, the following parameters of PID-type controller are received with the best value of the cost function.

θ_{opt} = {-0.132, -4.1.10^{-5}, -1.27, -6,4.10^{-5}, -1.45.10^{-2}, 7.3.10^{-5}} (39)

$$K_{PD1} = \frac{-0.132 - 4.1.10^{-5}s}{(0.0000318s+1)^2} \quad (40)$$

$$K_{PD2} = \frac{-1.27 - 6.4.10^{-5}s}{(0.0000318s+1)^2} \quad (41)$$

$$K_{PD3} = \frac{-1.45.10^{-2} + 7.3.10^{-5}s}{(0.0000318s+1)^2} \quad (42)$$

With the cost function as follows

$$J_2 = \beta\|E(s)\|_2^2 + \alpha\|U(s)\|_2^2 = 92.14$$

$$J_{\infty,a} = \|W_1(s)\,T(s)\|_\infty = 0.971$$

$$J_{\infty,b} = \|W_2(s)\,S(s)\|_\infty = 0.892$$

The simulation result of closed loop system with designed controller is shown in the Fig. 4 at working speed of Ω = 40.000 rpm.

4. Conclusion

The paper presents a process of designing a robust and optimal controller for active magnetic bearing systems. The active magnetic bearing systems are widely applied for high speed machining due to no contact operation, low fiction, etc. However, It is a non-linear, unstable, multiple input and multiple output system. Therefore, a robust and optimal controller is required. In the paper, we derive dynamic equation and analyze response of the open loop system. Based on the dynamic of the system, we propose a suitable controller with robust and optimal criteria using particle swarm optimization. The simulation results show that the closed loop system attains good performance in compared with conventional PID controllers.

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ĐIỀU KHIỂN TỐI ƯU VÀ BỀN VỮNG H_2/H_∞ CHO HỆ VÒNG BI TỪ TRƯỜNG TÍCH CỰC

Tóm tắt:
Bài báo này trình bày thuật toán thiết kế bộ điều khiển tối ưu và bền vững cho hệ vòng bi từ trường tích cực. Hệ vòng bi từ trường tích cực được sử dụng rộng rãi cho các hệ thống chuyển động tốc độ cao do hoạt động không tiếp xúc, ma sát thấp, không cần bôi trơn, và có tuổi thọ cao. Tuy nhiên, hệ này là hệ phi tuyến, không ổn định, có nhiều đầu vào và nhiều đầu ra. Bởi vậy, một bộ điều khiển tối ưu và bền vững là cần thiết. Trong bài báo này, chúng tôi xây dựng phương trình động lực học và phân tích đáp ứng của hệ vòng hở. Dựa trên đáp ứng đó, chúng tôi đề xuất một bộ điều khiển phù hợp với tiêu chuẩn tối ưu và bền vữngsử dụng tối ưu hóa bầy đàn. Kết quả mô phỏng chỉ ra rằng hệ vòng kín đạt chất lượng tốt hơn so với bộ điều khiển PID thông thường.
Từ khóa: *Bộ điều khiển bền vững, điều khiển H_∞/H_2, tối ưu hóa bầy đàn, hệ vòng bi từ trường, hệ MIMO.*